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Promoting Creativity for All Students in Mathematics Education

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WHICH KIND OF CREATIVITY MAY BE ATTAINED BY MOST STUDENTS?

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Abstract. *Most students may creatively developed only solutions to given problems, which should be achieved in the long run by utilizing suitable teaching/learning approaches that promote thinking skills related to creativity with powerful digital tools. Such an attained creativity may mostly be demonstrated at the so-called mild level where problems are solved by focusing on their details.*

We have often heard about the goal "mathematics for all" (obviously meaning mathematics for most students or almost all of them). A desired educational goal indeed, but, to the author's knowledge, there is no evidence that it has been attained, by showing that, for example, students in a random sample of the TIMSS eight grade classes can, on average, score higher on tasks of mathematics for all than they did on the TIMSS tasks (not only in statistical but also practical terms such as "a 5% difference level reached").

The things are much complex when we consider "creativity for all", because creativity, in general, demands good intellectual abilities involving analytical, creative and practical thinking skills (seeing issues in new ways beyond the conventional thinking; recognizing ideas that should be pursued; persuading others of the value of one's unconventional solution). Furthermore, "creativity requires a confluence of six distinct but interrelated resources: intellectual abilities, knowledge, styles of thinking, personality, motivation, and environment. Although levels of these resources are sources of individual differences, often the

decision to use the resources is the more important source of individual differences" (Sternberg, 2006; p. 6).

Having in mind that mathematics is based upon problem solving, creativity may be examined in terms of critical, creative and complex thinking. These types of thinking, their major thinking skills, and the underlying thinking skills of these major skills are examined in Jonassen (2000). Table 1 summarizes thinking skills by type of thinking. It can be said that creativity in mathematics is mostly manifested by a successful non-conventional problem solving that involves good critical and creative thinking.

Table 1. Major types of thinking skills and their underlying skills by type of thinking

Type of thinking	Major types of thinking skills and their underlying skills
Critical	**Evaluating**: assessing information, determining criteria, prioritizing, recognizing fallacies, and verifying. **Analyzing**: recognizing patterns, classifying, identifying assumptions, identifying main ideas, and finding sequences. **Connecting**: comparing/constraining, logical thinking, inferring deductively, inferring inductively, and identifying causal relationships.
Creative	**Elaborating**: expanding, modifying, extending, shifting categories, and concretizing. **Synthesizing**: analogical thinking, summarizing, hypothesizing, and planning. **Imagining**: fluency, predicting, speculating, visualizing, and intuition.
Complex	**Designing**: imagining a goal, formulating a goal, inventing a product, assessing a product, and revising the product. **Problem solving**: sensing the problem, researching the problem, formulating the problem, finding alternatives, choosing the solution, and building acceptance. **Decision making**: identifying an issue, generating alternatives, assessing the consequences, making a choice, and evaluating the choice.

What may the products of creative thinking in school mathematics be?

A created, mathematically correct object may be a problem solution most often, less often a method, and rarely a concept or a proved statement. Of course, though at a generalized level, the created mathematical object may also be an instructional design for teaching a mathematical topic. Some examples are given below.

- **Problem solution**. 5 is equal to $8-\frac{8+8+8}{8}$ or $\sqrt{8+8+8+\frac{8}{8}}$, but also to $\sqrt{8+8}+\left(\frac{8}{8}\right)^8$ or $\left(\sqrt{8+\frac{8}{8}}\right)!-\frac{8}{8}$.
- **Method**. Divide fractions by using rule $\frac{a}{b}:\frac{c}{d}=\frac{a:c}{b:d}$.
- **Concept**. Polynomial cx^2+bx+a is symmetric to polynomial ax^2+bx+c.
- **Proved statement**. If the roots of a quadratic polynomial are x_1 and x_2, then the roots y_1 and y_2 of its symmetric polynomial are $\frac{1}{x_1}$ and $\frac{1}{x_2}$. Suppose that x_1 and x_2 are the root of polynomial ax^2+bx+c. By applying the Vieta's rules, we get $x_1+x_2=-\frac{b}{a}\wedge x_1x_2=\frac{c}{a}$ as well as $y_1+y_2=-\frac{b}{c}\wedge y_1y_2=\frac{a}{c}$, yielding $y_1+y_2=\frac{1}{x_1}+\frac{1}{x_2}\wedge y_1y_2=\frac{1}{x_1x_2}$. If we assume that the solution of this system is $(y_1,y_2)=\left(\frac{1}{x_1}+t,\frac{1}{x_2}-t\right)$, we get $t^2=\frac{x_1-x_2}{x_1x_2}t$. The solutions of this equation are 0 and $\frac{x_1-x_2}{x_1x_2}$, which both result in $(y_1,y_2)=\left(\frac{1}{x_1},\frac{1}{x_2}\right)$.

Note that creativity may be found in problem posing if students demonstrate some kind of novelty with respect to context, content or solution strategy when defining tasks to be solved. However, a relationship between creativity and problem solving, if any, is not clear (see, for example, Leung, 1997).

Which of these types of objects may be created by students of average mathematics ability?

In the long run probably only creative problem solutions may be attained. In order to have the average students solve mathematical tasks in creative ways — a reachable goal in the light of the Japanese school mathematics (see Hiebert,

Stigler & Manaster, 1999) — we need to explain to students some general strategies for doing and accomplishing this task, as well as to, for easy references, arm them with appropriate sources about the knowledge and skills they have learned.

The first requirement — to explain to students some general strategies that help them solve mathematical problems in creative ways — should not be interpreted as a claim that creativity can be taught. If creativity can be taught, it will no more remain creativity. However, we can help students develop their critical, creative and problem solving skills by developing and using teaching/learning approaches that promote their underlying skills (see Table 1) and relate them in explicit ways.

The second requirement — to, for easy reference, arm students with appropriate sources about the knowledge and skills they have learned — is relevant to promoting creative solutions not only because the use of these resources considerably reduces the cognitive load of the given task. If these resources are in a digital format representing knowledge, skills, problem solving strategies and solution to selected problems, it is also relevant because the use of digital resources enables students to connect issues they would not connect without these resources, especially when the context of these resources can be searched in textual and semantic ways (see Barzilai & Zohar, 2006). Of course, suitable scaffolding in exploiting these digital resources in such a way should be developed and applied.

Textual and semantic searches are attained by using search tools such as Google. Apart from search tools, there are other powerful digital tools today, which, according to Jonassen (2000), can be used for *semantic organization* (e.g. databases and concept maps tools), *dynamic modelling* (e.g. spreadsheets and microworlds), *interpretation* (e.g. search tools and visualization tools), *knowledge construction* (e.g. hypermedia and multimedia tools), and *conversation* (e.g. asynchronous and synchronous conferencing tools). To provide more opportunities for learning, different kinds of these tools should be integrated. For example, an automated theorem prover can be integrated with a geometric microworld in such a way that properties, suggested by experiments on constructed objects, can be not only formally verified (e.g. WinGCLC at www.emis.de/misc/software/gclc/), but also added with their illustrations and proofs to a digital knowledge repository for later uses (Janicic & Quaresma, 2006).

Undoubtedly, developing thinking skills required by creativity should be based upon the use of such versatile digital tools. By using them student should first and foremost learn to view mathematical objects from different perspectives (e.g. for different views of derivative, see Thurston, 1998), which is an important necessary condition for creativity. In order to achieve this end, some mathematical learners may make use of learning through hypermedia/multimedia instructional design and express their creativity in developing such complex mathematical objects (see Kadijevich, 2004).

It is important to underline that creativity in problem solving can, according to Livne, Livne and Milgram (1999), be expressed at mild, moderate or profound level. At the mild level students solve tasks by focusing on details, whereas at the moderate level they achieve that by integrating underlying knowledge. At the profound level students solve tasks through finding solutions of less general or more general problems or examining solutions of problems in other (not necessarily mathematical) domains and applying these solutions to solve the original tasks (see Pólya, 1990). It is thus reasonable to expect that, in the long run, most students would be able to demonstrate creativity on the mild level, less at the moderate level and much less at the profound level. Three solutions that reflect these levels of creativity are given below.

- **Mild level**. Is $\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}} > 3$? **Solution**: $\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}} <$ $\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5+4}}}} = < \sqrt{5+\sqrt{5+\sqrt{5+3}}} < \sqrt{5+\sqrt{5+\sqrt{5+4}}}, \ldots,$ $\sqrt{5+\sqrt{5+\sqrt{5+\sqrt{5}}}} < 3$. The answer is negative.
- **Moderate level**. The coordinates of the vertices of a triangle are $A(-5, -2)$, $B(5, 1)$ and $C(2, 5)$. What is the measure of its angle BAC in degrees? **Solution**: If point A is translated into the origin, the rays of that angle can be represented by complex numbers $10 + 3i$ and $7 + 7i$. By multiplying them we get number $91 + 91i$. The answer is 45 degrees.
- **Profound level**. What triangle inscribed in an ellipse has the largest area? **Solution**: An equilateral triangle inscribed in a circle has the largest area. Apply a parallel projection to this circle and the equilateral triangle. Take into account the invariants of parallel projections. To have the largest area, the centroid of a triangle inscribed in an ellipse is to coincide with the center of that ellipse.

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To summarize: Mathematical objects that can be creatively developed by most students may primarily be problem solutions. This should be achieved in the long run by utilizing suitable teaching/learning approaches that make use of digital tools that promote thinking skills related to creativity. Such an attained creativity may be mostly demonstrated at the so-called mild level where problems are solved by focusing on their details. Further research may deal with the theoretical issues of different types of mathematical creativity, the practical issues of technology-based approaches supporting these types, and the empirical values of these approaches that would hopefully convince us that "creativity for most" is still attainable.

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GIFTED AND NON-GIFTED STUDENTS AND TEACHERS

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Abstract. Some very subjective remarks on the meaning, role and task of gifted or talented persons in general education. We gathered some experiences in a comparative geometry project between plane and sphere at various levels of education. We conclude that mathematical creativity could be found and developed further in the overwhelming majority of students if we created an educational environment that gave way to independent thinking and decision making ability. We try to give reasons why comparative geometry can create such environment by its mental and physical tools.

Key words: gifted and non-gifted students; extraordinary mathematical talent, comparative geometry between plane and sphere; talent and responsibility

MY STANDPOINT AND BACKGROUND EXPERIENCES

I am an educator and researcher whose professional work has been connected with very diverse types of age groups and background knowledge, from six-year-olds to tertiary education and teacher training; from laymen among the audience of a science museum to highly specialized mathematics researchers. My familiarity with a given age group is little as compared with that of an