XXX
Simpozijum o operacionim istraživanjima
SYM-OP-IS 2003
Hotel PLAŽA, Herceg-Novi, 30.09.-03.10.2003.

BASIC REQUIREMENTS FOR THE DESIGN AND ASSESSMENT OF A COMPUTER-BASED COURSE ON MATHEMATICAL MODELLING

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Abstract: *On the basis of research concerning teaching and learning mathematical modelling, this paper presents five requirements that should be respected whenever a computer-based course on mathematical modelling is being designed or evaluated. Despite their relevance, these requirements, which deal with epistemological, educational and computational issues of mathematical modelling, have so far rarely been implemented in the educational settings.*

KEY WORDS: MODELLING, INSTRUCTIONAL DESIGN, TERTIARY EDUCATION

0 INTRODUCTION

Despite various educational benefits, the inclusion of modelling and applications in mathematics education has typically been justified by a utilitarian argument [1], which may result in an inappropriate design and assessment of modelling course and its outcome. On the basis of research concerning teaching and learning mathematical modelling, this paper presents five requirements that should be respected whenever a computer-based course on mathematical modelling is being designed or evaluated. Despite their relevance, these requirements, which deal with epistemological, educational and computational issues of mathematical modelling, have so far rarely been implemented in the educational settings.

1 UTILIZE A HUMANISTICALLY-ORIENTED CONTEXT OF MODELLING

Mathematics has been applied to almost all areas of human endeavour. Although the scientificy of a particular field is typically measured by the degree to which its theories can be successfully mathematized, mathematizations in economics, medicine, sociology, politics and other fields are based upon a problematic assumption that solutions to problems in those fields can always be found by suitable quantification and computation. It should be made clear that, no matter how good it is, a mathematical model cannot annul an inappropriate data quantification, nor can it justify the societal consequences of its application based upon such a quantification. Two quotations that may further guide our thinking about the limits and validity of modelling are given below.

Whenever mathematics is employed to recommend or to set in motion a certain course of action, it is done on the basis of a criterion. One never knows the validity of the criterion nor does one know the full implication of putting a particular criterion to work. ... In computerized world economics, in computerized medicine, who sets the criteria and on what basis? ... *Quis custodet custodis* - who will watch the watchmen?

We must have more education for scientists in the humanities, in history. We cannot afford to be ignorant technicians. We must have less rigidity of thought. We must avoid becoming a scientific priesthood. ... We must remember that though Mathematics may be the Queen of the Sciences, Science is not the only principle of life. "The tree of life is larger than the tree of thought," said Immanuel Kant. [2, pp. 75, 85, 86]

2 PRESENT MODELLING AS A COMPLEX PROCESS

Although professional modelling is a complex *process* dealing with objects, properties and relations that develop and change over time as our understanding of the underlying problem progresses, textbooks rarely present several incrementally-developed models concerning the same real life situation. Even in simple examples like the break-even analysis — given a, b, and c, for what x is $ax + b$ (total production cost) equal to cx (sale revenue) ? — we can and should refine an initial model by introducing tax, depreciation, pessimistic and optimistic calculation variants, and so on (Fig. 1).

BREAK-EVEN ANALYSIS

Cost Variables	Per Month Optimistic -20%	Per Month Average Budgeted	Per Month Pessimistic 20%
Fixed Costs			
Sales & Marketing	$80	$100	$120
G & A (without Depreciation)	$200	$250	$300
Depreciation	$0	$0	$0
(other fixed costs)	$240	$300	$360
Total Fixed Costs	$520	$650	$780

	Per Month Optimistic -20%	Per Month Average Budgeted	Per Month Pessimistic 15%
Variable Costs			
Cost of Goods Sold	$2,088	$2,610	$3,002
(other variable cost)	$0	$0	$0
Total Variable Costs	$2,088	$2,610	$3,002

Pricing & Unit Sales Variables	Per Month Optimistic 25%	Per Month Sales Estimate	Per Month Pessimistic -20%
Selling Price	$30.00	$30.00	$30.00
Number of Units Sold	125	100	80
Fixed Costs per Unit	$4.16	$6.50	$9.75
Variable Costs per Unit	$16.70	$26.10	$37.52
Break-Even Unit Volume	39	167	-104
Units over Break-Even Unit Volume	86	-67	184
Gross Profit per Unit	$13.30	$3.90	($7.52)
Gross Profit (over Break-Even)	$1,142	($260)	($1,382)

Fig. 1 A screenshot of an Excel model developed by Richard Hyland from Hyland Enterprises

Modelling can be adequately described by using, for example, the following 3-stage framework:

1. *formulation* (formulate the problem; make an assumption in the model (set up a model); and formulate the mathematical problem);
2. *solution* (solve the mathematical problem); and
3. *application* (interpret the obtained solution; validate the model; and use the model to explain, predict, decide or design).

These seven steps are presented in Fig. 2.

Fig. 2 Seven modelling steps (see [3, p. 14])

This description primarily presents an experienced modelar's view of this complex task. But modelling involves various people (actors) and the final product typically reflects the interactions among them, which, because of different ways of thinking, different values, etc., put an additional complexity to the whole enterprize. Although basically two kind of actors are involved: *builders* who are building a model and *users* who have ordered the model and are applying (or will apply) it, professional modelling may involve four kind of actors: a *modelar* who builds a model, a *project manager* who provides resources for the modelling process, a *decision-maker* who will use the model to support decision-making, and a *client* who will be served by the developed model. An estimation says that to develop an institutionalized model integrated into the ongoing process of organization requires 10-100 times more effort than to develop a prototype model [4].

3 USE MODELLING TO EMPOWER THINKING AND LEARNING

Mental models are internal knowledge representations that are, despite their implicitness, incompleteness, imprecision and incoherence with scientifically accepted knowledge (of course from an expert's point of view), useful personal tools for the interaction of the subject with the world. Conceptual models, on the other hand, are community-shared knowledge representations that are precise, complete and coherent with scientifically accepted knowledge A common belief is that, as a result of teaching, students develop/acquire/construct mental models that are copies of conceptual models presented to them. However, a great number of studies in mathematics and science education evidence that this is not the case. Instead of developing adequate mental models mirroring the presented conceptual models or making somewhat hybrid models, students can often memorize the presented conceptual models and use them in school/academic settings, whereas they exploit their mental models in all informal settings (during their formal education and after it when most of the learned conceptual models are already forgotten). How to help students develop the desired mental models eventually? A promising tool is offered by modelling, whereby mental models can incrementally be developed in the direction of the desired conceptual models [5].

A technologist may say "Everything is in principle computation provided that good models are developed – *Computo, ergo sum*." A humanist may reply "Develop and use models, but don't loose the meaning of knowledge – *Comprehensio, ergo sum.*" Having taken a balanced view of the power of modelling, a person concerned with the teaching/learning of modelling may require "Use modelling primarily to empower thinking and conceptualize the problem areas more clearly – *Modelo, ergo comprehensio*."

4 RECOGNIZE AND EMPOWER COGNITIVE, METACOGNITIVE AND AFFECTIVE ISSUES OF MODELLING

From the psychological/didactical point of view, learning/teaching modelling is a very complex enterprize. This is because building a model is based upon a demanding interplay of modelar's cognitive, metacognitive and affective domains, and perhaps also because it is based upon complex interactions with other modelling actors (players) whose thinking, attitudes, beliefs, preferences, values, skills, etc. may be quite diverse.

Why is building a model based upon such an interplay?

As regard mathematical problem solving, Schoenfeld [6] finds that this process can be effectively analyzed by using the following five categories: (1) the knowledge base; (2) problem-solving strategies (heuristics); (3) selfregulation, or monitoring and control; (4) beliefs and affect; and (5) classroom practice, and that problem solving performance can be explained by interactions among student's cognitive, metacognitive and affective domains dealing with these categories. Modelling, as a specific instance of problem solving, is also influenced by such interactions, probably in a more demanding way than in pure mathematical problem solving.

Which of cognitive, metacognitive and affective issues seem particularly relevant to the process of modelling?

Concerning cognitive and metacognitive issues, a particular attention should be paid to the process of mathematization: clarify a real problem, generate variables, select variables, and set up conditions. This process requires three types of thinking: (1) clarifying vague conditions if any; (2) determining whether those variable affect the real solution and, if so, to what extent they do; and (3) setting up those conditions that enable an easy (easier) solution to the mathematical problem [7]. As regards affective domain, a particular attention should be paid to the student's belief systems regarding the mathematical domain and the problem domain and other affective issues since they determine a context (positive, negative or neutral) within which his/her cognitive and metacognitive activities are utilized (see [8]).

Modelling is firmly based upon mathematics. Many students are not good at mathematics. To avoid a negative affective context regarding mathematics, two things should be recognized: (1) mathematics can be conquered by hard work; and (2) this hard work is rewording as the knowledge of mathematics empowers our thinking. These things are illustrated in the next two sections.

- A summary of a report regarding the NCTM Standards for School Mathematics (see www.nctm.org) stresses, among others, the following issue:

  > .. in order to grow intellectually, students must have significant intellectual demands placed upon them. The 1989 Standards emphasized the responsibility of our profession to stimulate and "mathematically empower" students, but they did not simultaneously emphasize the necessity for students to work hard and stretch their attention spans. Students need to be aware that mathematics is not an inborn inherited trait and that everyone finds it difficult at some stage. They need to realize that in order to succeed they must not stop even when discouraged or frustrated. [9, p. 17]

  Of course, there are also emotional, socialized, spiritual and other kinds of personal development that are as important as the intellectual one, but, without a challenge, at a reasonable level of difficulty, no kind of development is likely to occur.

- Conrad Hilton, the founder and owner of the Hilton hotels in the United States and around the world (some 170 hotels during the 1950's), was one of the great businessmen of his time. Did he like mathematics? A quote below speaks for itself.

  > I'm not out to convince anyone that calculus, or even algebra and geometry, are necessities in the hotel business. But I will argue long and loud that they are not useless ornaments pinned onto an average man's education. For me, at any rate, the ability to formulate quickly, to resolve any problem into its simplest, clearest form, has been exceedingly useful. It is true that you do not use algebraic formulae but in those three small brick buildings at Sacorro I found higher mathematics the best possible exercise for developing the mental muscles necessary to this process.
  >
  > In latter years I was to be faced with large financial problems, enormous business deals with as many ramifications as an octopus has arms, where bankers, lawyers, consultants, all threw in their particular bit of information. It is always necessary to listen carefully to the powwow, but in the end someone has to put them up all together, see the actual problem for what it is, and make a decision - come up with an answer. A thorough training in the mental disciplines of mathematics precludes any tendency to be fuzzy, to be misled by red herrings, and I can only believe that my two years at the School of Mines helped me to see quickly what the actual problem was - and where the problem is, the answer is. Any time you have two times two and *know* it you are bound to have four. [10, pp. 63-64]

5 USE COMPUTERS AS MINDTOOLS FOR MODELLING

> *By employing technology as widely as possible, we can dedicate enough time to teach the choosing of model and the translating. Once these skills are taught explicitly, more students will appreciate and master them.* — Bernhard Kutzler

Modelling should utilize able programs such as Microsoft Excel or Texas Instruments Derive. But no matter which software is used, the "black box" view of the applied tool should, whenever possible, be avoided, by giving/requiring conceptual explanations of the performed calculations (Fig. 3).

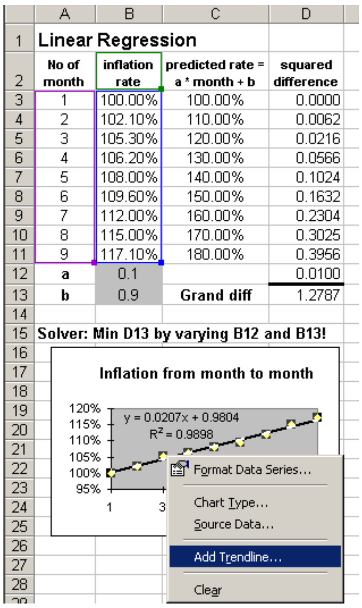

Fig. 3 What is behind *Add Trendline* in Excel?

ISTE Educational Technology Standards for Students (www.iste.org) require learners to use able programs as tools for communication, research, problem-solving, decision-making, and productivity, all of which are relevant to the process of modelling. If learners are required to develop models through Internet or other database searches, (e-)communications with the teacher and other students (modelling actors) involving Web/slides-based presentations of the developed models and accompanying courses of actions, computers will indeed be used as mindtools [11] promoting better thinking and learning.

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