Towards relating procedural and conceptual knowledge by CAS

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Abstract. The degree to which mathematics learning is successfully attained, depends on the degree to which learner can successfully cope with the coordination of different mathematical entities (competences, activities, knowledge types, representations, etc.). In order to promote links between procedural and conceptual knowledge by CAS, learners should be required to (1) relate mathematics in question to available CAS techniques and vice versa; and (2) solve problems in a process way as well as an object way by coordinating different representations. The fulfilment of these two requirements is illustrated for tasks from the upper secondary education concerning functions, equations and inequalities. These requirements help learners' use CAS in a functional and strategic way. In order to use CAS in a pedagogical way as well, learning mathematics through multimedia design may be applied, where multimedia lessons can be realized in the form of HTML files linking text, pictures, screen shots, video clips captured by a screen recorder, etc. Having in mind difficulties in promoting P-C links caused by CAS as well as potential obstacles in learning mathematics through multimedia design, teachers may scaffold their students through instruction that makes use of minimalism. Directions for further research are included.

INTRODUCTION

Shortcommings of mathematics education

Various evaluations around the world indicate that students' knowledge of mathematics has usually not been acquired to a degree expected by educators and employers (see, for example, Kadijevich, 2004). Such an unfavourable state is, for example, supported by the TIMSS 2003 Grade 8 results:

- in 34 of 46 participating countries, less than 30% of the tested students reached the so-called high international benchmark referring to the ability to apply understanding and knowledge to a wide variety of relatively complex situations;
- in 39 of 46 participating countries less than 10% of the tested students reached the so-called advanced international benchmark characterized by the ability of organize information, make generalization, solve non-routine problems, and draw and justify conclusions from data (see Mullis *et al*, 2004).

Reasons for unfavourable mathematics education

Mathematical proficiency has many interwoven and interdependent competences such as problem posing and solving, modelling, reasoning, representing, handling symbols and formalisms, communicating, and utilizing tools and aids (Blomhøj & Jensen, 2003) or problem solving, reasoning & proof, communication, connections and representation (NCTM, 2000). Also, various interlinked motives and activities such as play, calculate, apply, construct, evaluate, argue, find and order have contributed to the philogenesis of mathematical knowledge (Zimmermann, 2003). Despite that, mathematics teachers have tended to focus on one aspect at a time, wrongly hoping that other aspect(s) would (spontaneously) develop as a consequence (see Kilpatrick & Swafford, 2002). And, even when teachers use good problems (e.g. those that focus on concepts and connections among mathematical ideas), they can implement them in a wrong or inadequate way, e.g. as problems that call for basic computational skills and procedures (see Stigler & Hiebert, 2004). Major challenges of mathematics education are thus related to developing and linking different competences as well as to solving problems through relating the underlying conceptual and procedural knowledge.

Approach to improve the matters

Multimedia is a powerful tool for knowledge construction. Because those who learn more from the instructional materials are their developers, not users (Jonassen, 2000), mathematical learners should design multimedia lessons. In doing that computers should be used as mindtools i.e. constructivist partners that "engage learners in representing, manipulating, and reflecting on what they know" (Jonassen, 2000, p. 10). Bearing in mind the change in the view of mathematical knowledge from a hierarchical to a networked structure (inspired by Burton, 1999), learning through multimedia design should promote a flexible network nature of mathematical knowledge.

The ISTE educational technology standards requires students to use educational technology as versatile tools for productivity, communications, research, problem solving and decision making (see http://cnets.iste.org/currstands/cstands-netss.html). This requirement is highly relevant to mathematics learning through multimedia design. Kilpatrick (1992) requires solvers to effectively communicate what they have done with assigned tasks and the medium in question is very suitable for such a purpose. Also, when problem solving is supported by a tool such as CAS (Computer Algebra System) or DGS (Dynamic Geometry Software), this learning can fulfil the rest of the above-mentioned requirement referring to research, problem solving and decision making. By taking such an extensive technology-supported approach, learning mathematics through multimedia design would involve *problem solving*, *designing* and *decision making*, which are the three main activities involving complex thinking skills (Jonnasen, 2000).

Critical issues of that approach

Learning through multimedia design can be a rewarding learning experience not only for future teachers (see Bari & Gagnon, 2003; Kadijevich & Haapasalo, 2004; Eskelinen, 2005) but also for middle school students (Liu & Hsiao, 2002). Although research on multimedia should move from evaluation to theory-driven research (Moore, Burton & Mayers, 2004), the literature suggests many critical issues that are likely to affect the success of mathematical learning through multimedia design. Some of them are given below.

- Through engaging learners in solving real-world problems that are personally meaningful to them, multimedia design managers should make use of an activation-demonstration-application-integration approach to knowledge to be learned (Merrill, 2002), where the integration part (demonstrating the acquired knowledge publicly, reflecting on it, developing and exploring new ways of its use, etc.) may play the most prominent role.
- Guided by carefully developed learning principles (an initial set can be found in Mayer, 2001; and Kadijevich & Haapasalo, 2004), multimedia designers should realize and implement a network of learning paths being aware of their opportunities and limitations (presented to the users of their products in some suitable ways). One of these principles should require the learner to coordinate different mathematical entities (competencies, activities, knowledge types, views of knowledge, object representations, etc.).
- Because of the network nature of mathematical knowledge, whenever unsupported by research and practice, multimedia design managers should assume that mathematical entities (competences, representations, etc.) usually develop iteratively, one influencing the other(s). Such a state seems to

apply to procedural and conceptual mathematical knowledge (see Haapasalo, 2003; Rittle-Johnson & Koedinger, 2004). Thus instead of focusing whether entity *A* comes before *B* or vice versa, learners should rather be continuously guided to make use of and profit from a back-and-forth movement between *A* and *B* (extrapolated from Heid, 2002).

- Having in mind that learners typically have persistent orientations to learning that may be more innate than acquired (e.g., a preference for meaning, reproducing or achieving orientation found in Entwistle, 1988; a predisposition towards descriptive or relational images reported by Pantazi & Gray, 2000; a preference for from-concepts-to-skills, from-skills-to-concepts or skills-only approach suggested by Järvelä, 2003), multimedia design managers should be aware of these orientations, continuously demonstrate to designers how extreme orientations to learning may limit it, and encourage them to revise their conceptions of learning and of themselves as learners.
- Although technology based tools (usually computers i.e. software executed on them) should primarily be viewed in the Vygotskian sense as tools that expands our mental functions, students may, nevertheless, hold various views promoting different kinds of learning (e.g. technology as a master, servant, partner or an extension of self; see Galbraith, 2002). Because of that, multimedia design managers should be aware of designers' views on the utilized tools and help them overcome extreme views limiting their learning bearing in mind that not only opportunities and limitations of a tool affect the thinking of its user, but also that through getting to know a tool, the user more and more personalizes its usage (see Guin, Ruthven & Trouche, 2005).

RELATING PROCEDURAL AND CONCEPTUAL MATHEMATICAL KNOWLEDGE

As already underlined, relating procedural and conceptual mathematical knowledge (or mathematical skills and understanding in terms of Nesher (1986) or process and object features of mathematical knowledge in the terminology of Sfard (1994) or thinking proceptually in the approach of Grey and Tall (1993)) is a major challenge of mathematics education. Has this challenge been properly addressed so far in the context of technology supported learning?

Although some fifteen year ago Kaput asked "how different technologies affect the relation between procedural and conceptual knowledge" (Kaput, 1992; p. 549), CAL studies that examine the effects of their treatments regarding the co-ordination of procedural and conceptual mathematical knowledge are very rare. Furthermore, their results are opposing: while links between procedural and conceptual knowledge (P-C links) could be established in some studies, no such links were found in the others (see Kadijevich & Haapasalo, 2001). As regards the CAS context, while Hochfelsner & Kligner (1998) did not find that P-C links were established, Zehavi (1997) suggests that through replicating partially solved tasks (i.e. resolving tasks whose partial CAS solutions are given) these links can be promoted. In order to help learner establish P-C links, we need to better understand means by which these two knowledge types seem to be related.

Basic means to promote P-C links

According to Peschek and Schneider (2001), mathematics can be viewed as the science of making and updating links among representing, operating and interpreting, where technology supported mathematics learning allows more time for representing, interpreting and reflecting on the three and their relations. If we take representing to be the foundation of understanding in school mathematics, the central (yet research underrepresented) issues are related to selecting, using, and coordinating different representations (Heid, 2002). Relating different representations can not only support the development of conceptual knowledge (Papert, 1987), but also relate procedural and conceptual knowledge (Schwarz, Dreyfus & Bruckheimer, 1990; Haapasalo, 2003; Haapasalo & Kadijevich, 2003;

Haapasalo, Zimmermann & Rehlich, 2004). Because of that, although summarized in a somewhat simplified way, it can be said that in order to relate procedural and conceptual mathematical knowledge (or the process and object features of mathematical knowledge, or to think proceptually), multiple forms of representation are to be utilized and connected, especially with the aid of modern technological tools. Such a position is in accord with Rittle-Johnson, Siegler & Alibali (2001) who found that a change of problem representation influences the relation between the two knowledge types.

One-sided use of CAS

The transition from process to object is easier in the CAS learning environment than the traditional one (Gjone, 2004) because processes can be more easily manipulated as objects. For example, one expression can be inserted into the other on the selected place, a module representing a process can be used as an object, or algebraic and geometry representations of mathematical objects can be updated simultaneously as in the case of Casio ClassPad pocket calculator (see www.classpad.org). Despite these CAS affordances, it seems that learners in general rarely utilize CAS to coordinate the process and object features of mathematical knowledge. They rather tend to approach the given task in a process or an object way. For example, to determine a limit (e.g. $\lim \frac{\sqrt{x}+\cos x}{x+\sin x}, x \to +\infty$), the solver may utilize the standard CAS command, obtain "undefined" for the given expression as well as for some of its sub-expressions ($\sin x, \cos x$), and conclude that the function in question does not have limit because some of its sub-functions also do not have limit (see Trouche, 2005). Also, a limit for which CAS returns "undefined" may be re-approached in a process way (through sketching the graph of the underlying function and examining the obtained "indelicate" graph in detail) yielding an answer such as $x = 1.055$ and $y = 1.09 \cdot 10^{12}$ (see Gjone, 2004). These examples also show that the solver may not only persistently work in one representation (symbolic or graphic), but also demonstrate reasoning within CAS not without it. Because of all these issues (using a process or an object approach; working in one representation; reasoning within CAS), it can be said that students' use of CAS is usually one-sided.

Reasons for one-sided use of CAS

Such an unfavourable state has been influenced by students' mathematical knowledge and level of mastery of the utilized CAS environment (possibly perceived as an idiosyncratic tool, see Drijvers, 2000). According to Pierce and Stacey (2004), the degree to which CAS is used effectively depends on the degree to which its user can successfully cope with technical and personal challenges (being fluent with CAS syntax, changing CAS representations systematically, having no difficulties with CAS answers; holding positive CAS attitude, using CAS in strategic, functional and pedagogical way). Furthermore, the use of CAS offers more strategies for solving problems, which, as Brown (2003) underlines, introduce difficulties for both students and their teachers: while students have to reconcile different solution strategies, teachers have to find appropriate ways to assess more diverse solutions of their students. It is thus safer for CAS users to pursue one-sided approaches.

The one-sided use of CAS has also been influenced by inappropriate tasks (they rarely focus on relating procedural and conceptual issues) and missing explicit requirements for expected solutions concerning, for example, the work in different representations. If we require students to answer question "Does function $y = e^{\frac{1}{x}}$ have an extremum?" through relating process and object approaches (or procedural and conceptual knowledge), they may sketch the graph of this function by CAS, use it to demonstrate that this function is not defined for $x \neq 0$ resulting in answer “There is no extremum!”, and finally check this answer by using CAS to find the first derivative of the function ($y' = -\frac{1}{x^2} e^{\frac{1}{x}}$) and show that this derivative is less than zero for its entire domain.

Requirements to overcome one-sided use of CAS

In order to overcome the presented one-sided use of CAS concerning the coordination of the process and object features of mathematical knowledge, suitable requirements for CAS-based learning are to be developed and widely utilized. Bearing in mind the use of technology in applications and modelling in general, to promote appropriate reasoning that goes beyond the utilized technological tool, learning should respect the following requirement: When using mathematics, don’t forget available tool(s); when utilizing tool, don’t forget the underlying mathematics (Kadijevich, Haapasalo & Hvorecky, 2005). If, for example, one deals with the extreme values of the sine or cosine function (e.g. $d(t) = 8 + 0.7 \cdot \cos\frac{\pi}{6}t$), he/she would directly make use of the facts that $-1 \leq \sin x, \cos x \leq 1$ instead of relying on sketching the graph of the function in question and examining it in detail. Also, in order to solve system $x^2 + y^2 = 2x + 24 \wedge x + y = a$ (introducing parameters generates suitable CAS tasks, see Böhm *et al*, 2004), one should make use of some CAS built-in commands. Another requirement, based upon the considerations in the previous parts, is more specific: In order to solve the assigned task, use, whenever possible, a process approach as well as an object approach, working in multiple-registers (algebraic and graphical, for example). If, for example, an object approach to the limit in question results in status “undefined”, a process approach would suggest the value of that limit, whereas an object approach, which makes use of reasoning within mathematics (e.g. if $g(x) \leq f(x) \leq h(x)$, then $\lim g(x) \leq \lim f(x) \leq \lim h(x)$), would verify it. The relevance of the two requirements can be, for example, recognized in Meagher (2005) where the knowledge of the relationship between the maximum of a function and the sign of its first derivative helps students find the maximum of a function with a complex analytic expression during their work with different representations.

Designing CAS tasks that promote P-C links

When examining what role a CAS can play in problem solving, a brief answer is that it may be the main or an auxiliary tool, or just a tool that trivializes this process (see Lokar & Lokar, 2001). In order to find whether equation $e^x = \sqrt{x-3}$ has a solution, CAS can trivialize problem solving if the graphs of the corresponding functions are sketched or the underlying equation solved by using CAS built-in commands. If we require students to solve this problem in both conceptual and procedural ways with a “Justify your argument” prompt proposed by Brown (2003), they can use both ways mentioned in the previous sentence, but the sense of the trivialization of problem solving may still be present. This sense is removed if students use one “don’t forget” argument such as $e^x > x > \sqrt{x-3}$ (possibly validated by CAS).

Having in mind that a task whose natural solution requires an object approach may in a CAS environment be attempted by a process approach, and vice versa (Gjone, 2004), it is particularly important to insist on fulfilling the second requirement. It can be achieved even in tasks that do not look very promising at first sight. If, for example, an identity is to be confirmed (e.g. $\sin 3x = 3\sin x - 4\sin^3 x$), a process approach (several rows of eligible transformations) may be replaced by an object approach simply testing the equality of the two objects given on the left and right sides of that identity (see Gjone, 2004). However, the requirement to use both a process and an object approaches by working in more than one form of representation would improve the learning opportunities of this task considerably. This is because along with the transformations in question completed by CAS, the graphs of the underlying functions may be compared, and the equality of the two objects verified by solving an appropriate equation.

Difficulties in promoting P-C links caused by CAS

A competent use of a sophisticated technological device and the transition from tool (impersonal device) to instrument (personal device) is achieved through a long process of instrumental genesis (see Trouche, 2005), which is hindered by some features of CAS. According to Drijvers (2004), for simple instrumentation schemas such as *solve* and *substitute*, the process of instrumental genesis can extend the user's conceptual understanding (i.e. expressions can be "pasted" into variables; functions and expressions can be used as objects). However, for a more comprehensive scheme such as *isolate-substitute-solve* where more procedural knowledge is combined with conceptual knowledge, the success of this process in relating the two knowledge types seems to be mainly conditioned by the *congruence* between a paper-and-pencil technique and its CAS version and the *transparency* of that version. Drijvers (2004) underlines that these two conditioners need to be adequately address through class discussions and demonstrations.

Several researchers have underlined that CAS needs to be improved to become a pedagogical tool enabling technology-supported learning that is better than the traditional one without this tool (see Artigue, 2001). This has occurred not only because of inadequate congruence and transparency (Kutzler & Kokol-Voljc, 2003; see also Steiner & Dana-Picard, 2004), but also because there should be a possibility to distribute computational subtasks both to CAS and its user (Kutzler & Kokol-Voljc, 2003) and, when non-equivalent transformations are applied, CAS should require its user to declare the domain of the underlying function and, if applicable, warn him/her that some solutions are lost or surplus (Kadijevich, 2002). Furthermore, CAS should have a customizable user interface (Kutzler & Kokol-Voljc, 2003) to enable its users to master the tool in an easier way.

Approach to reduce these difficulties

The idea to have CAS with customizable user interface (customized by the teacher or the user himself/herself) may be viewed as an instance of minimalism – an approach to document and instructional design proposed by Carroll (1990, 1998). He advocates solving tasks that are personally meaningful and motivating in direct, solver-tailored ways with the smallest amount of the instructional verbiage. Minimalist instruction is based upon four principles: (1) approach is action-oriented, (2) tool is task-anchored, (3) errors are recognized and recovered, and (4) reading to do, study and locate is supported (Van der Meij & Carroll, 1998). Because of that, learning goals are derived from authentic tasks, learner needs determined by on-going assessment, learning modelled and coached on the spot, learning errors used as opportunities for learning, learning by doing and exploration encouraged resulting in multiple perspectives and solutions, and learning results assessed by the achieved transfer (Lambrecht, 1999). The use of the minimalist approach to instructional design—supported by appropriate video clip animations (e.g. www.joensuu.fi/lenni/classpad/example3.avi.zip)—may be a way to overcome or reduce some difficulties in promoting P-C links caused by CAS or other versatile technology tools (see Haapasalo, 2005).

CLOSING REMARKS

The degree to which mathematics learning is successfully attained depends on the degree to which learner can successfully cope with the coordination of different mathematical entities (competences, activities, knowledge types, representations, etc.). In order to promote P-C links by CAS, learners should be required to (1) relate mathematics in question to available CAS techniques and vice versa; and (2) solve problems in a process way as well as an object way by coordinating different representations. Further research may thus focus on the requirements for CAS supported solutions, which would help teachers adequately assess more diverse solutions of their students. Having in mind difficulties in promoting P-C links caused by CAS, teachers may scaffold their students through instruction that makes use of minimalism and further research may help teachers in doing that by refining the principles of such an instruction in the CAS context. One of these principles concerning the scaffolding of learning how to translate between representations may require teachers to start with dynamic linking provided by software, continue with signalling some of the connections between representations (gradually fading these connections), and end with making one representation reflecting another manually in the sense of the initial dynamic linking (Ainsworth, 1999).

Requirements (1) and (2) help learner use CAS in a functional and strategic way. In order to use CAS in a pedagogical way as well (recall Pierce & Stacey, 2004), learning mathematics through multimedia design may be applied, where multimedia lessons can be realized in form of HTML files linking text, pictures, screen shots, video clips captured by a screen recorder, etc. This kind of learning should be based upon sound multimedia design principles such as “Promote P-C links by fulfilling requirements (1) and (2)”. Further research may thus focus on the principles of learning mathematics through multimedia design that enhance mathematics learning and promote an adequate view of mathematics. Having in mind difficulties related to learning mathematics through multimedia design (see Kadijevich & Haapasalo, 2004; Haapasalo, 2005), multimedia design managers may, as in the case of promoting P-C links, help their designers through instruction that makes use of minimalism, whose principles may be adjusted to this kind of learning.

It must be underlined that most mathematics teachers do not realize the full power of computer assisted learning of mathematics (see Kadijevich, 2007), which prevents them from using CAS or other able technological tool in a way profitable to their learning and teaching (see Meagher, 2004). Bearing in mind that, as Artigue (1999) remarked, substantial improvements of the traditional teaching cannot be achieved by easy and inexpensive means without a strong institutional support and a substantial positive change in teachers’ knowledge, engagement, and day-to-day practice, P-C links are more likely to be promoted by CAS if this tool is given proper institutional and curricular values as requested by Pierce and Stacey (2004). Enthusiastic advocates of CAS-based mathematics learning should not forget that despite more then two decades of respectable international activities, mathematical modelling has had so far mostly a marginal role in everyday mathematics education at all educational levels (see Blum *et al*, 2002) because these values have not been given and approprite treatments utilized.

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