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DESIGN REQUIREMENTS FOR E-LEARNING OF MATHEMATICS: COMPLEXITY AND POSSIBLE SOLUTION

ZAHTEVI ZA DIZAJN E-UČENJA MATEMATIKE: KOMPLEKSNOST I MOGUĆE REŠENJE

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Abstract: *This paper examines the complexity of design requirements for e-learning in general and e-learning of mathematics in particular with respect to cognitive, metacognitive and affective learning issues. It also presents a web-based platform for learning mathematics that implements a part of this complexity in a very successful way.*

Keywords: *Design, E-learning, Mathematics*

Rezime: *Ovaj rad razmatra kompleksnost zahteva za dizajn e-učenja uopšte i e-učenja matematike posebno u odnosu na kognitivne, metakognitivne i afektivne osobenosti učenja. Rad takođe prikazuje web platformu za učenje matematike koja implementira deo prikazane kompleksnosti na veoma uspešan način.*

Ključne reči: *Dizajn, Elektronsko učenje, Matematika*

1. INTRODUCTION

Although sound instructional design principles of e-learning do not yet exist, this kind of learning should be based on the delivery of interactive multimedia content that respects the user's choice of the implemented states of learning (e.g. organizing, modeling, exploring) and the built-in navigational modes (i.e. learning strategies) that support the chosen learning state [1]. This instructional model is represented in Figure 1.

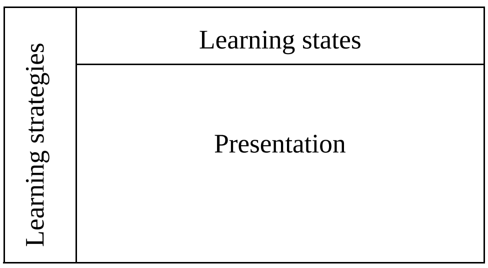

Figure 1: State-strategy-presentation model

Consider now first principles of instruction. They may be: (1) solve real-world problems, (2) use existing knowledge as a foundation for new knowledge, (3) demonstrate new knowledge to the learner, (4) apply new knowledge by the learner, and (5) integrate new knowledge in the learner's world [2]. Should these principles be used as learning states or learning strategies? Each of the two may be appropriate.

A model of e-learning enters into three or more dimensions when we include ways of learning and types of learning. Apart from teacher-directed learning (or guided learning), there are two ways of learning that are not controlled by a teacher or system [3]: experiential learning (when the learner learns though self-pursued activities) and action learning (when the learner learns through self-organized and self-planned learning). As regards learning types, the learner can learn about concept, procedure, or a network of concepts with associated procedures. Furthermore, he/she can learn through designing, problem solving or decision making, which puts into play underlying thinking skills (see Table 1 completed from [4]). Should then the learning strategies mentioned above promote these thinking skills? Even if we agree with all these facets of learning presented in this section, some of them may not be implemented in e-learning for technical or other reasons.

Table 1: Complex thinking skills and their sub-skills

Type	Sub-skills
Designing	Imagining a goal, formulating a goal, inventing a product, assessing a product, revising the product
Problem solving	Sensing the problem, researching the problem, formulating the problem, finding alternatives, choosing the solution, building acceptance
Decision making	Identifying an issue, generating alternatives, assessing the consequences, making a choice, evaluating the choice

Up to this point, we have presented the complexity of design requirements for e-learning in general. The next section examines this complexity with respect to mathematics learning, whereas the last section presents a web-based platform for learning mathematics that implements a part of this complexity in a very successful way.

2. COMPLEXITY

The learning of mathematics is constructive, self-regulated, both formal and contextual, and both individual and collaborative. To be competent in mathematics requires the acquisition of: (1) a domain specific knowledge base that is well-organized and flexibly accessible, (2) heuristics methods that improve problem solving, (3) meta-knowledge about one's productive cognitive and affective functioning, (4) positive mathematics attitude, and (5) self-regulatory skills regarding one's productive cognitive and affective functioning [3]. An important question here is how these cognitive, metacognitive and affective components can be expressed by design requirements.

The learning of mathematics may be examined in terms of mathematical competencies, which are suitable for the navigational modes examined above. One list of them comprises thinking, problem posing and solving, modeling, reasoning, representing, handling symbols and formulas, communicating, and using aids and tools [5]. Another list is shorter: just problem solving, reasoning & proof, communication, connections, and representation [6]. No matter which list is put into practice, competences should be cultivated in relation to each other (extrapolated from [7]), which, where appropriate, requires presenting links among built-in navigational modes.

Despite the popularity of the competence approach today, the learning of mathematics may be treated in other ways. One way is to foster key-activities for doing and creating mathematics that have successfully applied for several thousands years. Such activities are calculate, apply, construct, play, evaluate, argue, order, and find (see Fig. 2). Like mathematical competencies, these key-activities, linked to each other where appropriate, may be represented by the navigational modes.

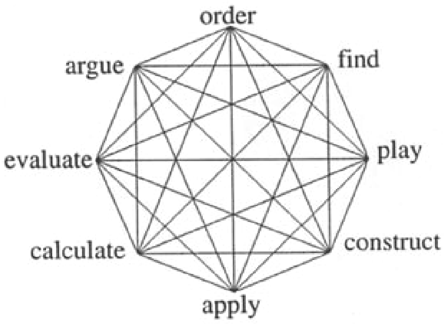

Figure 2: Activities that make mathematics ([8], p. 42])

Other way to treat the learning of mathematics is to explicitly deals with historical, epistemological, structural, and applicative issues of mathematical knowledge, and relate them [9]. Such a treatment of the subject opens a space for learning path – a way in which the learner „travels" through a particular mathematical topic to learn (about) it (see [10] for different paths).

Having in mind the previous discussion, we may imagine an e-learning generator that can personalize e-learning of mathematics according to the learner's preference for learning path, targeted competence, and instructional mode, such as:

- *learning path*: from applicative issues, to structural & epistemological issues, to historical issues (defined by available sequencing of the implemented global issues);
- *targeted competence*: making connections (chosen from available of the implemented competences);
- *instructional mode*: exploring i.e. examining different standpoints and their consequences (selected from a list of available strategies or components of the implemented instructional design).

Of course, available options for learning path, targeted competence and instructional mode (possibly constraining each other) may differ from topic to topic.

Apart from such a 3-D learning design, an improved version of the learning dashboard presented in Fig. 1 may have additional buttons concerning:

- *learning process* such as control (from learner to system), engagement (from passive to active), generativity (from presentation to creativity), see [4], scaffolding (from none to full), etc.
- *learning summaries* regarding the five learning components listed at the beginning of this section, the links between procedural and conceptual mathematical knowledge or the links among geometric, algebraic and abstract modes of representation, see [11, 12], etc.

Such a dashboard, sketched at Figure 3, reveals the complexity of design requirements (note that learning type is omitted), whose implementation seems to require an advanced web tool of the future. This is not true, however, as evidenced by the content of the last section.

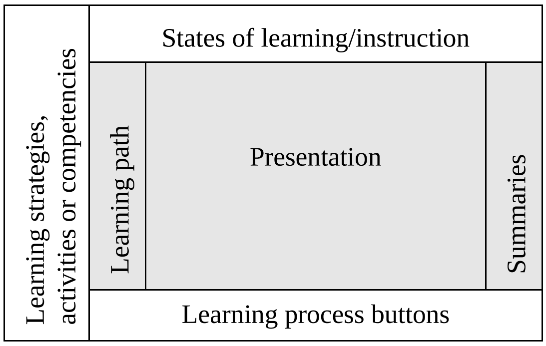

Figure 3: An improved e-learning dashboard

3. POSSIBLE SOLUTION

Complex design requirements presented in the previous section have been partially implemented in a web platform called *ActiveMath* [13]. This web-based, multi-lingual platform combines innovative approaches to e-learning and intelligent tutoring systems. Its main features are:

- courses are assembled according to learning goal, learning scenario, competence, learning content and preferences specified by the learner (or the teacher);
- tutoring is based upon the learner's model;
- the learner can explore his/her model generated by the system;

- the structures of the examined mathematical domains can be visualized by using an interactive concept mapping tool;
- the content of ActiveMath can be searched by using both text search and semantic search.

Figure 4 captures the details of one of steps in book creation. The learner (or the teacher) completes each step.

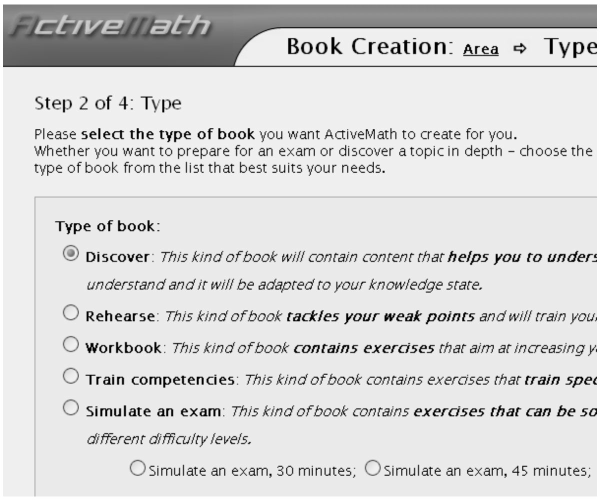

Figure 4: Step in book creation by ActiveMath

Figures 5 and 6 give the details of the learner's model generated by the system. As regards the data given in Fig. 6, one task was used to assess different competences.

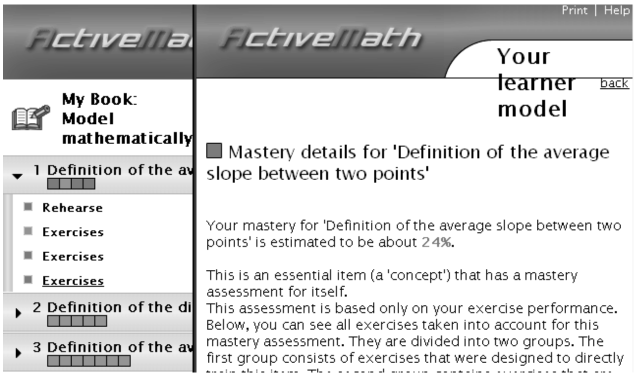

Figure 5: Mastery details for definition

In order to help teachers improve their teaching, ActiveMath has a tool called *student inspector*. It provides displays that show most frequent errors, students with the poorest performance, as well as weak and strong topics and competencies for a given student [14].

More details about the ActiveMath platform including its demo version can be found at www.activemath.org. This platform provides not only a possible solution to the complexity in question, but also a very promising one, which further research may improve with respect to the features of the learning dashboard presented in Figure 3.

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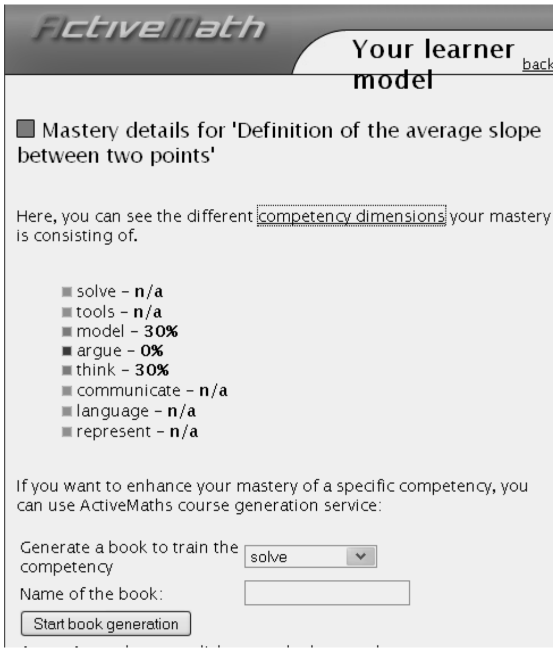

Figure 6: Mastery details for definition via competencies

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