CAS-BASED TASK REQUIREMENTS AND CRITICAL ACTIVITIES IN COMPLETING THEM *

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Abstract. Having in mind essential features of mathematics, appropriate CAS-based tasks should require representing, operating and interpreting with CAS. Critical activities in completing these tasks may be found in transitions from representing to operating, and from operating to interpreting. These critical activities—related to mathematics, technology or both—are, for example, using CAS to apply transformations of represented objects, and relating CAS results with mathematical questions aimed to be answered.

INTRODUCTION

According to Berger [1], critical design features of CAS-based tasks basically deal with constructing signs, using these signs, and interpreting the outcome of their use. She discusses these three features through consideration of an approximation of a function by a polynomial with a certain precision, suggesting that help in solving CAS-based tasks may be needed with respect to each of the features. In this reaction to [1] we wish to underline two important yet neglected issues. First, the three design features respectively deal with three kinds of mathematization: representing, operating and interpreting [2], each influencing the others. Second, by extrapolating from [3], critical activities in completing these kinds of mathematization with CAS should be found in transitions from representing to operating, and from operating to interpreting (intepreting with CAS without representing, and some operating with it, seems quite rare). In this account "CAS" does not denote just a symbolic algebra functionality but an integrated environment involving several tools (applications) supporting the work with several representations of mathematical entities.

REQUIREMENTS

Three basic activities in mathematics are representing, operating and interpreting [2]. As a result, mathematics can be viewed as the science of doing and relating these three kinds of mathematization, where using technology allows more time for representing and interpreting as well as for reflecting on the three mathematizations and their relations.

In addition to operating, technology can be used for representing (e.g., defining a function or generating a graph) and interpreting (e.g., visualizing an unfamiliar solution or searching a library of solutions of related problems). Because of that,

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CAS-based tasks should require the use of CAS for each of the three mathematizations. Easier tasks should focus on only one of them with CAS gradually adding the other(s) with focus both on them and their relations.[1] For example, we may start with representing the given problem situation with a piece-wise linear function, or with interpreting the solution of a corresponding equation (where the outcomes of representing and operating are provided). Then we may relate representing and operating to illustrate how a change in representing (e.g., from algebraic representation to graphical representation of piece-wise linear functions) may add complexity to or reduce it from operating. In general, using different representations may improve problem solving as well as understanding of the underlying mathematics as demonstrated in [4]. Although Berger [1] presents a useful framework to guide the design of CAS-based tasks, she does not describe what CAS affords with respect of each of the three mathematizations and relations among them.

CRITICAL ACTIVITIES

Critical activities in a transition from one modelling stage to the next are examined by Galbraith and Stillman [3]. As regards the transition from real world problem statement to mathematical model supported by technology, these researchers recognize the following nine critical activities:

1. Identifying dependent and independent variables for inclusion in algebraic model,
2. Realizing that independent variable must be uniquely defined,
3. Representing elements mathematically so formulae can be applied,
4. Making relevant assumptions,
5. Choosing technology/mathematical tables to enable calculation,
6. Choosing technology to automate application of formulae to multiple cases,
7. Choosing technology to produce graphical representation of model,
8. Choosing to use technology to verify algebraic equation,
9. Perceiving a graph can be used on function graphers but not data plotters to verify an algebraic equation. (See [3, p. 147].)

These activities describe not only what should happen when a particular transition is achieved with a success, but also what blockages are likely to cause a failure in that transition.

Modelling is one approach to doing mathematics. Having in mind the generality of the representing-operating-interpreting framework, this framework can be found in the modelling stages. Also, as underlined in the previous part, it is important to relate different kinds of mathematizations with CAS. Because of that, the presented approach of Galbraith and Stillman [3] may be extrapolated to transitions within the representing-operating-interpreting framework. In doing so, we tried to recog-

[1] Even tasks focusing on one critical aspect may be hard for students. Examples include tasks that ask one to “Sketch the graph of the given function on its whole domain”, and more demanding tasks that request one to “Find the function whose graph is given."

nize critical activities in transitions from representing to operating, and from operating to interpreting. Mostly relying on our experience in using CAS as a learning tool, we recognized that main critical activities in question may be (list is neither exaustive nor final):[2]

- **From representing to operating**: Using CAS to apply transformations of represented objects; Using CAS to verify a representation that will be used for calculation or interpretation; Using CAS to calculate.
- **From operating to interpreting**: Relating CAS results with mathematical questions to be answered; Preparing for interpreting through obtaining additional results with CAS; Preparing for interpreting through integrating all CAS results obtained.

Consider, for example, a task that requires solving the system of equations, $x + y = a$ and $x^2 + y^2 = 25$, in terms of parameter *a.* Assume that the user decided to use a parameter and two equations, chose to use an algebraic tool and a geometry tool, represented the parameter and the two equations with the algebraic tool, and represented the two equations with the geometry tool. The six critical activities listed above may be found in the following activities (the order of three within each transition is not fixed):

1. Compare the outcomes in the geometry window for different values of the parameter;
2. Produce a table of values for a relation in question and match it to the correponding graph produced by the geometry tool;
3. For concrete values of the parameter, find the solution of the system with the algebraic tool;
4. Recognize that the distance of the origin from the line (found first for concrete values of a) is related to the solvability of the system;
5. In terms of parameter *a,* find the solution of the system in question and the distance of the origin from line $x + y = a$ (a user-defined function may be used for the latter);
6. Integrate the results obtain under 5 having in mind related results obtained under 1 and 3.

In her account on designing CAS tasks, Berger [1] suggests that help in solving CAS-based tasks may be needed with respect to each of the three kinds of mathematization. Guidance for help in solving such tasks (i.e., scaffolding) – missing in her account – may profit from the presented critical activities, enabling teacher and researcher to manage better the design and use of CAS-based tasks.

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[2] Main crticial activities in transition from task statement to representing may be: Identifying objects to be used (functions, equations, inequalities, or others); Choosing CAS tool(s) to represent these objects; Representing identified objects with CAS.

CLOSING REMARKS

Appropriate CAS-based tasks should require representing, operating and interpreting with CAS, which would dynamically relate relevant conceptual and procedural knowledge (cf. [4]). Critical activities in completing these tasks, found in the two transitions mentioned above, are related to mathematics, technology or both. As students tend not to coordinate use of mathematics and use of relevant e-tools [5], particular attention should be paid to critical activities related to both mathematics and CAS combining exact mathematical language and math-jargon of technology [6].

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